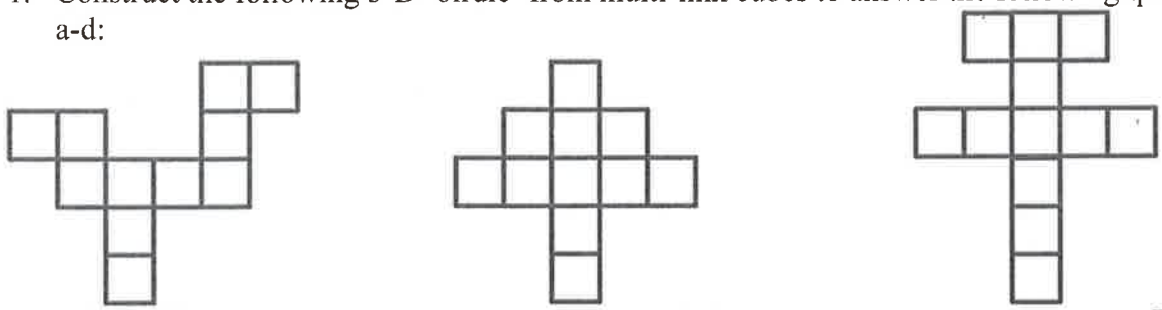
CI 161 TEST 2/Burger/Summer '14

Team: Solution Key

(20 pts)

1. Construct the following 3-D 'birdie' from multi-link cubes to answer the following questions a-d:



Side View   Front View   Top View

a. Find the Volume in u^3.

$17u^3$

b. Determine the Surface area in u^2.

$2 \cdot 11 = 22$
$2 \cdot 11 = 22$
$2 \cdot 12 = 24$ → 68 + 2 Hidden ⇒ $70u^2$

Behind neck and Front of tail.

c. How many cubes would be needed to ***triple the bird two times?***

Focus on 1 Block such as the Beak.
1 - tripling → 3 × 3 × 3 = 27 Blocks ← just in Beak)
second tripling → 9 × 9 × 9 = 729 Blocks
(17 • 729) → 12,393 Blocks!

tripling

V

$\left(\frac{1}{3}\right)^2 \cdot \left(\frac{70}{17}\right) = 0.46$

(0.4575)

(20 pts)

2. Prove by induction:

$n^3 - n$ is divisible by 3 for all natural numbers n.

$P(1)$: $1^3 - 1 = 0 = 3 \cdot 0$ ✓

Assume $k^3 - k = 3 \cdot l$

Need to prove $(k+1)^3 - (k+1) = 3 \cdot r$

Well:

$(k+1)^3 - (k+1) = k^3 + 3k^2 + 3k + 1 - k - 1$

remember 1 3 3 1 ?

But $k^3 - k = 3l$ so:

$k^3 + 3k^2 + 3k - k =$

$3l + 3k^2 + 3k =$

$3(l + k^2 + k) = 3 \cdot r$ ✓

(15 pts)

3. Find a vector perpendicular to the vectors $\langle 3, 2, 5\rangle$ and $\langle 2, -3, 1\rangle$ and show that this vector is perpendicular to 'one' of the above vectors.

$$\begin{matrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 3 & 2 & 5 & 3 & 2 \\ 2 & -3 & 1 & 2 & -3 \end{matrix} \rightsquigarrow (2\hat{i} + 10\hat{j} - 9\hat{k}) - (4\hat{k} - 15\hat{i} + 3\hat{j})$$

$$\rightarrow 2\hat{i} + 15\hat{i} + 10\hat{j} - 3\hat{j} - 9\hat{k} - 4\hat{k}$$

$$= 17\hat{i} + 7\hat{j} - 13\hat{k}$$

$$= \langle 17, 7, -13\rangle$$

$$\langle 3, 2, 5\rangle \cdot \langle 17, 7, -13\rangle = 17 \cdot 3 + 2 \cdot (7) + 5 \cdot (-13)$$

$$= 51 + 14 - 65$$

$$= 65 - 65 = 0$$

(5pts)

4. Construct a *unit vector* in the direction of the vector $\langle -8, 15\rangle$

$$\vec{u} = \frac{\langle -8, 15\rangle}{\|\langle -8, 15\rangle\|}$$

$$\|\langle -8, 15\rangle\| = \sqrt{(-8)^2 + 15^2} = \sqrt{64 + 225} = \sqrt{289} = 17$$

$$\therefore \vec{u} = \left\langle \frac{-8}{17}, \frac{15}{17} \right\rangle$$

(20 pts)

~~5~~ 4. Given $A = \begin{bmatrix} 1 & 4 & 3 \\ 1 & 3 & 5 \\ 2 & 7 & 7 \end{bmatrix}$, find A^{-1}.

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 1 & 3 & 5 & 0 & 1 & 0 \\ 2 & 7 & 7 & 0 & 0 & 1 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 1 & 0 \\ 0 & -1 & 1 & -2 & 0 & 1 \end{array}\right]$$

$-1 \cdot R2$

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 0 \\ 0 & -1 & 1 & -2 & 0 & 1 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 0 \\ 0 & 0 & -1 & -1 & -1 & 1 \end{array}\right]$$

$-R3$

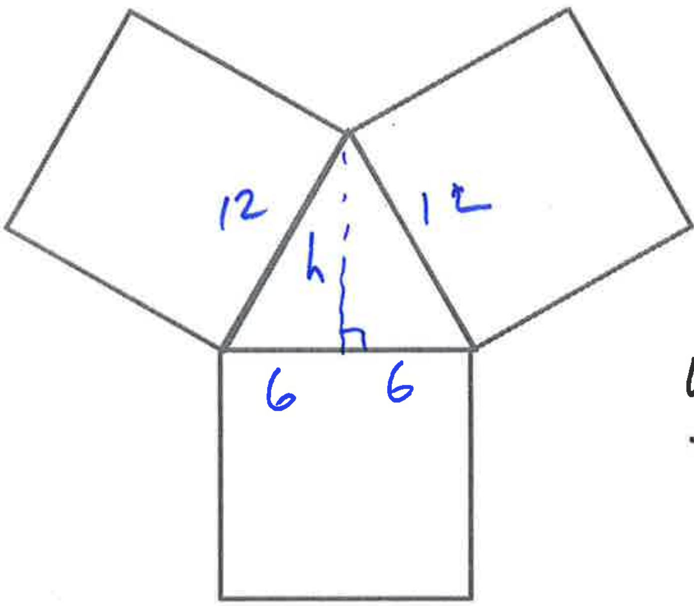
$$\left[\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 4 & 0 & -2 & -3 & 3 \\ 0 & 1 & 0 & 3 & 1 & -2 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -14 & -7 & 11 \\ 0 & 1 & 0 & 3 & 1 & -2 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array}\right] \rightarrow A^{-1} = \begin{bmatrix} -14 & -7 & 11 \\ 3 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix}$$

(20 pts)

RICH-PROBLEM SURPRISE!

5. The three squares below are of equal area. If the sum of the areas of the squares is 432 cm^2, what is the area of the triangle? ($A_\Delta = \frac{1}{2}bh$)



$\frac{432}{3} = 144\,cm^2$ each square

$\sqrt{144} = 12$ side of square

$6^2 + h^2 = 12^2$

$36 + h^2 = 144$

$h^2 = 144 - 36 = 108$

$h = \sqrt{108} = 6\sqrt{3}$

$b = 12$

$h = 6\sqrt{3}$

$A = \frac{1}{2} \cdot 12 \cdot 6\sqrt{3}$

Area of Triangle = $36\sqrt{3}\,cm^2 \approx 62.35\,cm^2$